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SWUP

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Welcoming Address

Welcome to the 2015 IConSSE – The International Conference on Science and Science Education!

This conference, which is organized by the Faculty of Science and Mathematics, Satya Wacana Christian University Salatiga, is held at Laras Asri Resort and Spa Salatiga.

Arts, science and technology are crucial components in the advancement of human civilization. There is art in the creation of technology, and science provides strong bases for the technological development. We are proud to inherit the temple of Borobudur which is a proof that Indonesian's ancient arts and technology are so advanced that not only is the masterpiece beautiful, but also technologically rich.

This International Conference on Science and Science Education is attended by more than 160 participants. There are more than 67 papers is presented orally covering wide-variety subjects of science and science education. We thank you all for your participation.

We thank the Organizing Committee, Reviewers, and Steering Committee for having been working hard. Finally, we would also like to thank the Rector of Satya Wacana Christian University, and Dean of Faculty of Science and Mathematics for their support for this conference.

We hope you will enjoy our togetherness. Thank you.

Salatiga, November 30th , 2015

Dr. Adi Setiawan
Chairman

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Simulation study of reliability coefficient and discrimination index

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Abstract

In this paper it will be presented how resampling method with replacement is used to construct a new simulated table and based on the simulated table is determined the reliability coefficient (by using Cronbach's alpha) and discrimination index (by using Ferguson's delta). Simulation study is done to find the relationship (is expressed by using Pearson correlation coefficient) between the discrimination index and reliability coefficient. We conclude that there exists a significant coefficient correlation between Cronbach's alpha reliability coefficient and Ferguson's δ discrimination index. The simulation study can also be done with a Likert scale in the next following research.

Keywords resampling, reliability coefficient, discrimination index, Ferguson's delta, Cronbach's alpha.

1. Introduction

In preparing the measurement tool widely used in psychology, reliability coefficient is very important. The reliability coefficient estimates the degree of measurement error in the data and hence the reproducibility of the measurements (Hankins, 2007). Discrimination index is used to distinguish between individuals without reference to an external criterion (Hankins, 2008). In this paper it will be presented how resampling method with replacement is used to construct a new simulated table and based on the simulated table is determined the reliability coefficient (by using Cronbach's alpha) and discrimination index (by using Ferguson's delta). Simulation study is done to find the relationship (is expressed by using Pearson correlation coefficient) between the discrimination index and reliability coefficient. Related recent papers related to this paper are Setiawan (2014a) and Setiawan (2014b).

2. Literature review and research methods

In the literature review it will be explained a method to calculate reliability coefficient by using Cronbach's alpha and discrimination index by using Ferguson's delta formula. Ferguson's original formula is appropriate for scales with dichotomous items:

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$$\delta = \frac{(k+1)\left(n^2 - \sum_{i=1}^{k} f_i^2\right)}{kn^2},$$

where k is the number of items, n is the sample size and f_i is the frequency of each score i (Hankins, 2007). To present an idea how to use the formula, the data is given in Table 1 which consists of 5 items of question answered by 10 examinees of the test (Terluin et al., 2009). A value of 1 means that the examinee responded correctly to an item and a value of 0 means the examinees responded incorrectly to the item. The data gives different total score $x = (1, 2, 3, 4, 5)$ with frequency $f = (2, 4, 1, 2, 1)$ respectively. By using Ferguson's delta formula, we have $k = 5$, $n = 10$ such that

$$\delta = \frac{(k+1)\left(n^2 - \sum_{i=1}^{k} f_i^2\right)}{kn^2} = \frac{(5+1)\left(10^2 - (2^2 + 4^2 + 1^2 + 2^2 + 1^2)\right)}{5(10)^2} = \frac{6(100-26)}{500} = 0.888$$

Table 3. Responses of 10 examiness to 5 items, dichotomously scored.

Examines	1	2	3	4	5	Total Score
1	0	0	1	1	0	2
2	0	0	0	1	0	1
3	1	1	0	0	0	2
4	1	0	0	1	0	2
5	0	1	1	1	1	4
6	0	1	0	0	0	1
7	1	1	1	1	1	5
8	1	1	0	1	0	3
9	1	1	1	1	0	4
10	0	0	0	1	1	2
Total	**5**	**6**	**4**	**3**	**3**	

In 1951, Cronbach presented a method to estimate the internal consistency with a formula that become known as Cronbach's Alpha. The alpha reliability coefficient was calculated by the formula

$$r = \left(\frac{k}{k-1}\right)\left(1 - \frac{\sum_{i=1}^{k} \sigma_i^2}{\sigma^2 x}\right),$$

where k specifies the number of items used in the calculations in the analysis, is the variance of the i-th item and is the total score variance (Siegel, 1956). Reliability coefficient is positive and significant means that the measurement tool is reliable otherwise the measurement tool is not reliable. Based on Table 1, the reliability coefficient by using Cronbach's alpha is 0.412.

For scales with more than two response options (such as Likert scales), the modified formula should be used :

$$\delta = \frac{\left(1 + k(m-1)\right)\left(n^2 - \sum_{i=1}^{k} f_i^2\right)}{kn^2(m-1)},$$

where m is the number of item responses (Hankins, 2007). To present an idea how to use the formula, the data is given in Table 2 which consists of 5 items of question answered by 10 respondents of the scale. A value of 5 means that the respondent answered strong agree to an item and a value of 1 means the respondent answered strong not agree to the item. The data gives different total score $x = (8, 9, 10, 14, 16, 17, 19, 22)$ with frequency $f = (1, 2, 1, 1, 1, 1, 2, 1)$, respectively. By using Ferguson's delta formula, we have $k = 5$, $n = 10$ and $m = 5$ such that

$$\delta = \frac{\left(1+k(m-1)\right)\left(n^2 - \sum_{i=1}^{k} f_i^2\right)}{kn^2(m-1)}$$

$$= \frac{(1+5(5-1))\left(10^2-\left(1^2+2^2+1^2+1^2+1^2+1^2+2^2+1^2\right)\right)}{5(10)^2(5-1)}$$
$$= \frac{21(100-14)}{2000} = 0.903$$

Based on Table 2, the reliability coefficient by using Cronbach's alpha is 0.808.

Table 2. Responses of 10 examiness to 5 items with score 1 through 5.

Examinees	1	2	3	4	5	Total Score
1	5	5	4	3	2	19
2	2	2	3	1	2	10
3	4	4	3	3	2	16
4	2	2	2	1	2	9
5	5	5	3	5	4	22
6	1	1	2	2	3	9
7	1	2	3	1	1	8
8	4	1	3	4	5	17
9	5	3	4	4	3	19
10	2	2	3	3	4	14
Mean	**3.1**	**2.7**	**3**	**2.7**	**2.8**	
Variance	**2.77**	**2.33**	**0.44**	**2.01**	**1.51**	

Research methods

Resampling method with replacement that is used in Table 1 can be explained as follows :

1) A simulated table can be obtained by using resampling with replacement every row of Table 1 and the Ferguson's formula is applied for the simulated table.
2) The procedure given in step 1 is repeated until big number *B* times such that *B* results of the Ferguson's delta formula are obtained i.e. $F_1, F_2, \ldots\ldots, F_B$
3) The procedure given in step 1 and step 2 can also be applied to reliability coefficient by using Cronbach's alpha such that it is obtained $C_1, C_2, \ldots\ldots, C_B$
4) The distribution of Ferguson's delta statistic and Cronbach's alpha can be determined by using $F_1, F_2, \ldots\ldots, F_B$ and $C_1, C_2, \ldots\ldots, C_B$, respectively. Mean, median and 95 % percentile confidence interval of discrimination index and reliability coefficient can be obtained.
5) Furthermore, the relation between discrimination index and reliability index can be determined by using Pearson correlation coefficient between $F_1, F_2, \ldots\ldots, F_B$ and $C_1, C_2, \ldots\ldots, C_B$.
6) Simulation studies are carried out with the following steps :
7) Data is in matrix form with order $m \times n$ and it is generated by a Bernoulli distribution with parameter p where p denote the probability of the examinees answered correctly for each item. There are m examinees and n items in the test. It is considered that the ability of examinees to answer each question independently. Based on the simulated data, it is calculated Ferguson's delta and Cronbach's alpha. If the procedure is repeated $B = 1000$ times and the Pearson correlation of coefficient between is calculated. If this way is repeated 100 times then we have $P_1, P_2, \ldots, P_{100}$ Pearson correlation coefficient and its mean. In this simulation study $m = 20,30,50,100$, while the number of test items $n = 20,30,50,100$. Furthermore, it is used $p = 0.1$ until $p = 0.9$ by using step 0.1.

8) Total score of examinees has a normal distribution $N(\mu, \sigma^2)$ such that if $X_1, X_2, \ldots, X_n$ are the total score then

$$Z_i = \frac{X_i - \bar{X}}{s}.$$

9) has a standard normal distribution where $\bar{X}$ and s are mean and standard deviation, respectively. Thus, the probability of examinee to answer every item correctly is $\Phi(Z_i)$ for $i = 1,2,\ldots,m$, where m is the number of examinees. In this simulation study $m = 20,30,50,100$, while the number of test items $n = 20,30,50,100$. In this case, we use μ = 65, σ = 10; μ = 65, σ = 5; μ = 60, σ = 10; μ = 60, σ = 5 ; μ = 55, σ = 10; μ = 65, σ = 5; μ = 50, σ = 10 and μ = 50, σ = 5.

10) Simulation that is used in this paper is Monte Carlo simulation.

3. Results and discussion

The simulated data (by using Monte Carlo simulation) is generated by using resampling as described before with replacement every row of Table 1. Table 3 presents the simulated data. Based on Table 3, it is obtained the Ferguson's delta and Cronbach's alpha are 0.912 and 0.792, respectively. Related paper to this method is (Setiawan, 2014a). If the procedure of simulated data is repeated in a big number $B = 1000$ and the histogram of *B* Ferguson's delta and Cronbach's alpha is presented in Figure 1. Figure 2 presents the relation between The Pearson correlation coefficient between Ferguson's delta and Cronbach's alpha values is 0.327 if the previous procedure is repeated in $B = 1000$ times.

Table 3. The simulated data based on Table 1.

Examinees	1	2	3	4	5	Total Score
1	0	0	0	0	1	1
2	0	1	0	0	1	2
3	1	0	1	0	1	3
4	0	1	1	0	0	2
5	1	1	1	1	1	5
6	1	0	0	0	0	1
7	1	1	1	1	1	5
8	0	0	1	0	0	1
9	1	1	1	1	1	5
10	0	0	0	0	0	0
Total	**5**	**5**	**6**	**3**	**6**	

Table 4 presents the relation (in Pearson coefficient of correlation) between Cronbach's alpha reliability coefficient and Ferguson's δ discrimination index for several *m*, *n* and *p* as a result of simulation study described in research method. We can conclude that there exist a significant coefficient correlation between Cronbach's alpha reliability coefficient and Ferguson's δ discrimination index. Furthermore, Table 5 presents the relation between reliability coefficient and discrimination index for several *m*, *n*, μ and σ. Similar result can be concluded for the relation between reliability coefficient and discrimination index. The result in this paper is similar with relation between item validity index and item discrimination index (Setiawan, 2014).

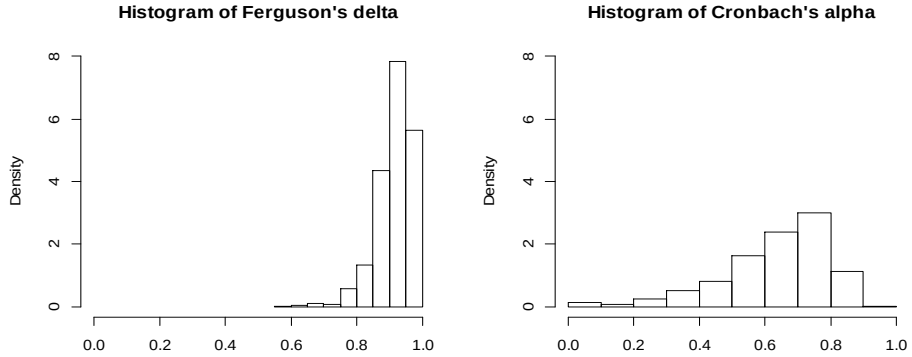

Figure 1. Histogram of $B = 1000$ Ferguson's delta values (left) and Cronbach's alpha values (right).

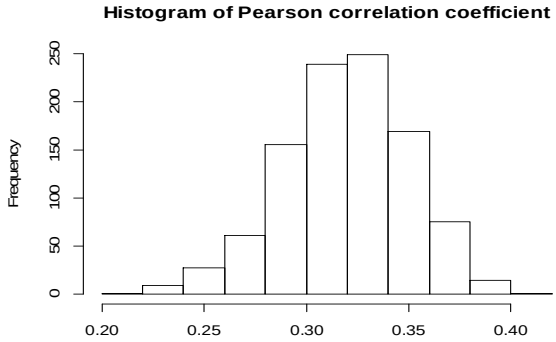

Figure 2. Histogram of $B = 1000$ Pearson Correlation Coefficient value between Ferguson's delta and Cronbach's alpha values.

Table 4. Result of simulation : the relation (in Pearson correlation coefficient) between reliability coefficient and discrimination index for several m, n, and p.

n	m	$p = 0.1$	$p = 0.2$	$p = 0.3$	$p = 0.4$	$p = 0.5$	$p = 0.6$	$p = 0.7$	$p = 0.8$	$p = 0.9$
20	20	0.66	0.65	0.68	0.67	0.67	0.67	0.67	0.68	0.65
	30	0.67	0.75	0.75	0.75	0.76	0.76	0.75	0.75	0.67
	50	0.67	0.76	0.78	0.77	0.77	0.77	0.77	0.76	0.67
	100	0.69	0.80	0.83	0.83	0.84	0.83	0.83	0.80	0.68
30	20	0.69	0.65	0.63	0.63	0.63	0.63	0.64	0.65	0.66
	30	0.70	0.73	0.73	0.72	0.75	0.72	0.73	0.73	0.70
	50	0.71	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.72
	100	0.74	0.80	0.81	0.81	0.81	0.82	0.81	0.80	0.74
50	20	0.68	0.61	0.59	0.57	0.57	0.57	0.58	0.60	0.64
	30	0.71	0.70	0.69	0.69	0.68	0.68	0.68	0.70	0.71
	50	0.73	0.72	0.71	0.71	0.71	0.71	0.72	0.72	0.73
	100	0.77	0.79	0.79	0.79	0.78	0.78	0.79	0.79	0.77
100	20	0.64	0.54	0.51	0.50	0.50	0.51	0.51	0.54	0.59
	30	0.68	0.64	0.62	0.62	0.61	0.61	0.62	0.64	0.69
	50	0.70	0.67	0.65	0.66	0.64	0.64	0.66	0.67	0.70
	100	0.77	0.76	0.75	0.75	0.74	0.74	0.75	0.76	0.77

Table 5. Result of simulation: the relation between reliability coefficient and discrimination index for several *m*, *n*, μ and σ.

n	*m*	$\mu = 65$, $\sigma = 5$	$\mu = 65$, $\sigma = 5$	$\mu = 60$, $\sigma = 10$	$\mu = 60$, $\sigma = 5$	$\mu = 55$, $\sigma = 5$	$\mu = 55$, $\sigma = 10$	$\mu = 50$, $\sigma = 10$	$\mu = 50$, $\sigma = 5$
20	20	0.38	0.38	0.28	0.30	0.31	0.38	0.38	0.33
	30	0.35	0.36	0.26	0.33	0.34	0.36	0.31	0.34
	50	0.32	0.32	0.21	0.33	0.32	0.31	0.31	0.32
	100	0.28	0.25	0.16	0.28	0.28	0.24	0.25	0.27
30	20	0.33	0.45	0.47	0.39	0.36	0.47	0.25	0.37
	30	0.36	0.43	0.42	0.38	0.38	0.44	0.28	0.39
	50	0.38	0.40	0.36	0.38	0.38	0.40	0.39	0.38
	100	0.34	0.31	0.28	0.34	0.33	0.31	0.31	0.33
50	20	0.36	0.53	0.67	0.34	0.40	0.57	0.55	0.43
	30	0.41	0.53	0.62	0.37	0.44	0.55	0.54	0.46
	50	0.43	0.49	0.55	0.38	0.45	0.50	0.49	0.46
	100	0.41	0.41	0.45	0.34	0.41	0.41	0.42	0.42
100	20	0.38	0.68	0.85	0.41	0.44	0.68	0.65	0.47
	30	0.44	0.68	0.82	0.46	0.49	0.68	0.66	0.52
	50	0.49	0.65	0.77	0.50	0.52	0.55	0.63	0.54
	100	0.49	0.57	0.68	0.50	0.51	0.57	0.56	0.52

Table 4 presents the relation (in Pearson coefficient of correlation) between Cronbach's alpha reliability coefficient and Ferguson's δ discrimination index for several *m*, *n* and *p* as a result of simulation study described in research method. We can conclude that there exist a significant coefficient correlation between Cronbach's alpha reliability coefficient and Ferguson's δ discrimination index. Furthermore, Table 5 presents the relation between reliability coefficient and discrimination index for several m, n, μ, and σ. Similar result can be concluded for the relation between reliability coefficient and discrimination index. The result in this paper is similar with relation between item validity index and item discrimination index (Setiawan, 2014).

4. Conclusion and remarks

In this paper it has been presented simulation study to find the relation between the discrimination index and reliability coefficient. We conclude that there exists a significant coefficient correlation between Cronbach's alpha reliability coefficient and Ferguson's δ discrimination index. The simulation study can also be done with a Likert scale in the next following research.

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